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A note on the existence and uniqueness of mild solutions to neutral stochastic partial functional differential equations with non-Lipschitz coefficients

Feng Jiang, Yi Shen *

Department of Control Science and Engineering and The Key Laboratory of Ministry of Education for Image Processing and Intelligent Control, Huazhong University of Science and Technology, Wuhan, 430074, China

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ABSTRACT

In this note, we study the existence and uniqueness of mild solutions to neutral stochastic partial functional differential equations under some Carathéodory-type conditions on the coefficients by means of the successive approximation. In particular, we generalize and improve the results that appeared in Govindan [T.E. Govindan, Almost sure exponential stability for stochastic neutral partial functional differential equations, Stochastics 77 (2005) 139–154] and Bao and Hou [J. Bao, Z. Hou, Existence of mild solutions to stochastic neutral partial functional differential equations with non-Lipschitz coefficients, Comput. Math. Appl. 59 (2010) 207–214].

1. Introduction

Stochastic partial differential equations in a separable Hilbert space often model some evolution phenomena arising in physics, biology, engineering, etc. [1]. There are many papers on the existence and uniqueness of solutions. For the case where the coefficients satisfy the global Lipschitz condition and the linear growth condition, many results are known [1,2]. However, the global Lipschitz condition, even the local Lipschitz condition, is seemed to be considerably strong when one discusses variable applications in the real world. Bao and Hou [3] discussed the existence of mild solutions to neutral stochastic partial functional differential equations with non-Lipschitz coefficients.

We are concerned with neutral stochastic partial functional differential equations in the case where the coefficients do not necessarily satisfy the global Lipschitz condition. Thus we discuss the existence and uniqueness of mild solutions to neutral stochastic partial functional differential equations with the condition proposed by the author [4]. This condition was investigated by Turo and Cao et al. [5,6] as a type of the Carathéodory condition for the strong solutions. Motivated by the above papers, in this note, we extend the existence and uniqueness of mild solutions to Eq. (1) under some Carathéodory-type conditions to Hilbert spaces, with the Lipschitz condition in [2] and the non-Lipschitz condition in [3] being regarded as special cases.

The rest of this note is organized as follows. In Section 2, we introduce some preliminaries. In Section 3, we prove the existence and uniqueness of the mild solution.

* Corresponding author.
E-mail addresses: jeff20@163.com (F. Jiang), yishen64@163.com (Y. Shen).

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2. Preliminaries

Throughout this note, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete probability space with a normal filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is increasing and right-continuous while $\mathcal{F}_0$ contains all P-null sets). Moreover, let X and Y be two real separable Hilbert spaces; we denote by $\langle\cdot,\cdot\rangle_X$, $\langle\cdot,\cdot\rangle_Y$ their inner products and by $\|\cdot\|_X$, $\|\cdot\|_Y$ their vector norms, respectively. We denote by $L(Y, X)$ the space of all bounded linear operators from Y into X, equipped with the usual operator norm $\|\cdot\|$. In this note, we always use the same symbol $\|\cdot\|$ to denote norms of operators regardless of the spaces potentially involved when no confusion possibly arises. Let $\tau > 0$ and $C = C([-\tau, 0]; X)$ denote the family of all continuous X-valued functions η from $[-\tau, 0]$ to X with norm $\|\eta\|_C = \sup_{t\in[-\tau,0]} \|\eta(t)\|_X$. Let $C^b_{\mathcal{F}_0}([-\tau, 0]; X)$ be the family of all almost surely bounded, $\mathcal{F}_0$-measurable, $C([-\tau, 0]; X)$-valued random variables.

Let $\{w(t) : t \geq 0\}$ denote a Y-valued Wiener process defined on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ with covariance operator Q; that is, $E\langle w(t), x\rangle_Y E\langle w(s), y\rangle_Y = (t \wedge s)\langle Qx, y\rangle_Y$, for all $x, y \in Y$, where Q is a positive, self-adjoint, trace class operator on Y. In particular, we denote $w(t)$ a Y-valued Q-Wiener process with respect to $\{\mathcal{F}_t\}_{t\geq 0}$. To define stochastic integrals with respect to the Q-Wiener process $w(t)$, we introduce the subspace $Y_0 = Q^{1/2}Y$ of Y endowed with the inner product $\langle u, v\rangle_{Y_0} = \langle Q^{-1/2}u, Q^{-1/2}v\rangle_Y$ as a Hilbert space. We assume that there exists a complete orthonormal system $\{e_i\}$ in Y, a bounded sequence of nonnegative real numbers λ_i such that $Qe_i = \lambda_i e_i$, $i = 1, 2, \ldots$, and a sequence $\{\beta_i(t)\}_{i\geq 1}$ of independent standard Brownian motions such that

$$w(t) = \sum_{i=1}^{\infty} \sqrt{\lambda_i}\beta_i(t)e_i, \quad t \geq 0$$

and $\mathcal{F}_t = \mathcal{F}_t^w$, where $\mathcal{F}_t^w$ is the σ-algebra generated by $\{w(s) : 0 \leq s \leq t\}$. Let $L_2^0 = L_2(Y_0, X)$ be the space of all Hilbert–Schmidt operators from Y_0 to X. It turns out to be a separable Hilbert space equipped with the norm $\|u\|^2_{L_2^0} = tr((uQ^{1/2})(uQ^{1/2})^*)$ for any $u \in L_2^0$. Obviously, for any bounded operator $u \in L_2^0$, this norm reduces to $\|u\|^2_{L_2^0} = tr(uQu^*)$.

Suppose that $\{S(t), t \geq 0\}$ is an analytic semigroup with its infinitesimal generator A; for literature relating to semigroup theory, we suggest Pazy [7]. We suppose that $0 \in \varrho(A)$, the resolvent set of $-A$. For any $\alpha \in [0, 1]$, it is possible to define the fractional power $(-A)^\alpha$, which is a closed linear operator with its domain $\mathcal{D}((-A)^\alpha)$.

In this work, we consider the following neutral stochastic partial functional differential equations (NSPFDEs):

$$\begin{cases} d[x(t) - u(t, x_t)] = [Ax(t) + f(t, x_t)]\mathrm{d}t + g(t, x_t)\mathrm{d}w(t), & t \geq 0, \\ x_0(\cdot) = \varphi \in C^b_{\mathcal{F}_0}([-\tau, 0]; X), \end{cases} \tag{1}$$

where $x_t = \{x(t+\theta) : -\tau \leq \theta \leq 0\}$ can be regarded as a $C([-\tau, 0]; X)$-valued stochastic process. $u : R_+ \times C([-\tau, 0], X) \to X$, $f : R_+ \times C([-\tau, 0], X) \to X$, $g : R_+ \times C([-\tau, 0], X) \to L(Y, X)$ are all Borel measurable; A is the infinitesimal generator of an analytic semigroup of bounded linear operators $S(t)$, $t \geq 0$, in X.

Definition 2.1. A process $\{x(t), t \in [0, T]\}$, $0 \leq T < \infty$, is called a mild solution of Eq. (1) if

(i) $x(t)$ is adapted to $\mathcal{F}_t$, $t \geq 0$ with $\int_0^T \|x(t)\|_X^2 \mathrm{d}t < \infty$ a.s.;
(ii) $x(t) \in X$ has continuous paths on $t \in [0, T]$ a.s., and for each $t \in [0, T]$, $x(t)$ satisfies the integral equation

$$\begin{aligned} x(t) &= S(t)[\varphi(0) - u(0, \varphi)] + u(t, x_t) + \int_0^t AS(t-s)u(s, x_s)\mathrm{d}s \\ &\quad + \int_0^t S(t-s)f(s, x_s)\mathrm{d}s + \int_0^t S(t-s)g(s, x_s)\mathrm{d}w(s), \end{aligned} \tag{2}$$

for any $x_0(\cdot) = \varphi \in C^b_{\mathcal{F}_0}([-\tau, 0]; X)$.

To guarantee the existence and uniqueness of a mild solution to Eq. (1), the following much weaker conditions, instead of the non-Lipschitz condition, are used.

(H1) A is the infinitesimal generator of an analytic semigroup of bounded linear operators $S(t)$, $t \geq 0$, in X and $S(t)$ is uniformly bounded, for some constant $0 < a \in R_+$, $\|S(t)\| \leq e^{-at}$, $t \geq 0$.

(H2) (a) There exists a function $H(t, r) : R_+ \times R_+ \to R_+$ such that $H(t, r)$ is locally integrable in $t \geq 0$ for any fixed $r \geq 0$, and is continuous monotone nondecreasing and concave in r for any fixed $t \in [0, T]$. Moreover, for any fixed $t \in [0, T]$ and $\xi \in X$, the following inequality is satisfied:

$$\|f(t, \xi)\|_X^2 + \|g(t, \xi)\|^2_{L_2^0} \leq H(t, \|\xi\|_C^2), \quad t \in [0, T]. \tag{3}$$

(b) For any constant $K > 0$, the differential equation

$$\frac{\mathrm{d}u}{\mathrm{d}t} = KH(t, u), \quad t \in [0, T], \tag{4}$$

has a global solution for any initial value u_0.

(H3) (a) There exists a function $G(t, r) : R_+ \times R_+ \to R_+$ such that $G(t, r)$ is locally integrable in $t \geq 0$ for any fixed $r \geq 0$, and is continuous monotone nondecreasing and concave in r for any fixed $t \in [0, T]$. $G(t, 0) = 0$ for any fixed $t \in [0, T]$. Moreover, for any fixed $t \in [0, T]$ and $\xi, \eta \in X$, the following inequality is satisfied:

$$\|f(t, \xi) - f(t, \eta)\|_X^2 + \|g(t, \xi) - g(t, \eta)\|_{L_2^0}^2 \leq G(t, \|\xi - \eta\|_C^2), \quad t \in [0, T]. \tag{5}$$

(b) For any constant $\overline{K} > 0$, if a nonnegative function $z(t)$ satisfies that

$$z(t) \leq \overline{K} \int_0^t G(s, z(s))\mathrm{d}s, \quad t \in [0, T], \tag{6}$$

then $z(t) = 0$ holds for any $t \in [0, T]$.

(H4) The mapping $u(t, x)$ satisfies that there exists a number $\alpha \in [0, 1]$ and a positive K_0 such that, for any $\xi, \eta \in X$ and $t \geq 0$, $u(t, x) \in \mathcal{D}((-A)^\alpha)$ and

$$\|(-A)^\alpha u(t, \xi) - (-A)^\alpha u(t, \eta)\|_X \leq K_0\|\xi - \eta\|_C.$$

Moreover, we assume that $u(t, 0) = 0$.

Remark 2.1. Let $G(t, u) = L(t)\overline{G}(u),\ t \in [0, T]$, where $L(t) \geq 0$ is locally integrable and $\overline{G}(u)$ is a concave nondecreasing function from R_+ to R_+ such that $\overline{G}(0) = 0$, $\overline{G}(u) > 0$ for $u > 0$ and $\int_{0+} \frac{1}{\overline{G}(u)}\mathrm{d}u = \infty$. Then, by the comparison theorem of differential equations we know that assumption (H3-b) holds.

Now let us give some concrete examples of the function $\overline{G}(\cdot)$. Let $\zeta > 0$ and let $\delta \in (0, 1)$ be sufficiently small. Define

$$\overline{G}_1(u) = \zeta u, \quad u \geq 0$$
$$\overline{G}_2(u) = \begin{cases} u \log(u^{-1}), & 0 \leq u \leq \delta, \\ \delta \log(\delta^{-1}) + \overline{G}_2'(\delta-)(u - \delta), & u > \delta, \end{cases}$$

where $\overline{G}_2'$ denotes the derivative of function $\overline{G}_2$. They are all concave nondecreasing functions satisfying $\int_{0+} \frac{1}{\overline{G}_i(u)}\mathrm{d}u = \infty$ ($i = 1, 2$). In particular, we see that the Lipschitz condition in [2] and the non-Lipschitz conditions in [3] are special cases of our proposed condition.

To show our main results, we need the following lemma.

Lemma 2.1 ([7]). *If* (H1) *holds and* $0 \in \varrho(A)$, *then, for any* $\beta \in (0, 1]$,

(i) *for each* $x \in \mathcal{D}((-A)^\beta)$, $S(t)(-A)^\beta x = (-A)^\beta S(t)x$;
(ii) *there exist positive constants* $M_\beta > 0$ *and* $a \in R_+$ *such that* $\|(-A)^\beta S(t)\| \leq M_\beta t^{-\beta} e^{-at},\ t > 0$.

3. Existence and uniqueness of the solution

In this section, we establish the existence and uniqueness theorem of the mild solution.

Theorem 3.1. *If* (H1)–(H4) *hold for some* $\alpha \in (1/2, 1]$, *then there exists a unique solution to Eq.* (1), *provided that*

$$\gamma := \frac{4K_0^2 M_{1-\alpha}^2 a^{-2\alpha} \Gamma(2\alpha - 1)}{1 - K_0\|(-A)^{-\alpha}\|} + K_0\|(-A)^{-\alpha}\| < 1, \tag{7}$$

where $M_{1-\alpha}$ *is defined in Lemma 2.1.*

Proof. To obtain the existence of the solution to Eq. (1), let $x^0(t) = S(t)\varphi(0),\ t \in [0, T]$ and $x_0^n(t) = \varphi(t),\ n = 0, 1, 2, \ldots,$ for $t \in [-\tau, 0]$, define the following successive approximating procedure: for each integer $n = 1, 2, \ldots,$

$$\begin{aligned} x^n(t) - u(t, x_t^n) = {} & S(t)[\varphi(0) - u(0, \varphi)] + \int_0^t AS(t - s)u(s, x_s^n)\mathrm{d}s \\ & + \int_0^t S(t - s)f(s, x_s^{n-1})\mathrm{d}s + \int_0^s S(t - s)g(s, x_s^{n-1})\mathrm{d}w(s). \end{aligned} \tag{8}$$

The proof is divided into the following three steps.

Step 1. We claim that the sequence $\{x^n(t),\ n\geq 0\}$ is bounded. From (8), for $0\leq t\leq T$,

$$\begin{aligned}E\sup_{0\leq s\leq t}\|x^n(s)-u(s,x^n_s)\|^2_X &\leq 4E\sup_{0\leq s\leq t}\|S(s)[\varphi(0)-u(0,\varphi)]\|^2_X+4E\sup_{0\leq s\leq t}\left\|\int_0^s AS(s-r)u(r,x^n_r)\mathrm{d}r\right\|^2_X\\ &\quad+4E\sup_{0\leq s\leq t}\left\|\int_0^s S(s-r)f(r,x^{n-1}_r)\mathrm{d}r\right\|^2_X+4E\sup_{0\leq s\leq t}\left\|\int_0^s S(s-r)g(r,x^{n-1}_r)\mathrm{d}w(r)\right\|^2_X\\ &=:4\sum_{i=1}^4 I_i.\end{aligned}\tag{9}$$

By (H1),

$$I_1\leq(1+K_0\|(-A)^{-\alpha}\|)^2E\|\varphi\|^2_C.\tag{10}$$

Note that

$$E(\sup_{0\leq s\leq t}\|x^{n-1}_s\|^2_C)\leq E(\sup_{0\leq s\leq t}\|x^{n-1}(s)\|^2_X)+E\|\varphi\|^2_C.$$

Applying the Hölder inequality, (H4) and Lemma 2.1, we have

$$\begin{aligned}I_2&\leq E\sup_{0\leq s\leq t}\left(\int_0^s\left\|(-A)^{1-\alpha}S(s-r)(-A)^\alpha u(r,x^n_r)\right\|_X\mathrm{d}r\right)^2\\ &\leq E\sup_{0\leq s\leq t}\left(\int_0^s M_{1-\alpha}e^{-a(s-r)}(s-r)^{\alpha-1}\left\|(-A)^\alpha u(r,x^n_r)\right\|_X\mathrm{d}r\right)^2\\ &\leq K_0^2M^2_{1-\alpha}a^{-2\alpha}\Gamma(2\alpha-1)E(\sup_{0\leq s\leq t}\|x^n(s)\|^2_X+\|\varphi\|^2_C).\end{aligned}\tag{11}$$

By (H2) and the Jensen inequality, we obtain

$$I_3\leq T\int_0^t H(r,E(\sup_{0\leq u\leq r}\|x^{n-1}(u)\|^2_X+\|\varphi\|^2_C))\mathrm{d}r.\tag{12}$$

By (H2), Liu [1, Theorem 1.2.6, p. 14] and the Jensen inequality, there exists a positive constant C_1 such that

$$I_4\leq C_1\int_0^t H(r,E(\sup_{0\leq u\leq r}\|x^{n-1}(u)\|^2_X+\|\varphi\|^2_C))\mathrm{d}r.\tag{13}$$

Recall that, for $a,b\in X$, $\varepsilon\in(0,1)$, $\|a-b\|^2_X\leq 1/(1-\varepsilon)\|a\|^2_X+1/\varepsilon\|b\|^2_X$. Hence, substituting (10)–(13) into (9) yields

$$\begin{aligned}E(\sup_{0\leq s\leq t}\|x^n(s)\|^2_X+\|\varphi\|^2_C)&\leq\frac{1}{1-K_0\|(-A)^{-\alpha}\|}E\sup_{0\leq s\leq t}\|x^n(s)-u(s,x^n_s)\|^2_X\\ &\quad+\frac{1}{K_0\|(-A)^{-\alpha}\|}E\sup_{0\leq s\leq t}\|u(s,x^n_s)\|^2_X+E\|\varphi\|^2_C\\ &\leq\frac{4(1+K_0\|(-A)^{-\alpha}\|)^2}{1-K_0\|(-A)^{-\alpha}\|}E\|\varphi\|^2_C+E\|\varphi\|^2_C\\ &\quad+\left(\frac{4K_0^2M^2_{1-\alpha}a^{-2\alpha}\Gamma(2\alpha-1)}{1-K_0\|(-A)^{-\alpha}\|}+K_0\|(-A)^{-\alpha}\|\right)E(\sup_{0\leq s\leq t}\|x^n(s)\|^2_X+\|\varphi\|^2_C)\\ &\quad\times\frac{4(T+C_1)}{1-K_0\|(-A)^{-\alpha}\|}\int_0^t H(r,E(\sup_{0\leq u\leq r}\|x^{n-1}(u)\|^2_X+\|\varphi\|^2_C))\mathrm{d}r.\end{aligned}\tag{14}$$

Assumption (H2-b) indicates that there is a solution u_t that satisfies

$$u_t=C_2E\|\varphi\|^2_C+C_3\int_0^t H(r,u_r)\mathrm{d}r,$$

where $C_2=\frac{1}{1-\gamma}(\frac{4(1+K_0\|(-A)^{-\alpha}\|)^2}{1-K_0\|(-A)^{-\alpha}\|}+1)$, $C_3=\frac{4(T+C_1)}{(1-\gamma)(1-K_0\|(-A)^{-\alpha}\|)}$.

Since $E\|\varphi\|^2_C<\infty$, from (14), we have $E(\sup_{0\leq s\leq t}\|x^n(s)\|^2_X)\leq u_t\leq u_T<\infty$, which shows the boundedness of the $\{x^n(t),\ n\geq 0\}$.

Step 2. We claim that $\{x^n(t),\ n\geq 0\}$ is a Cauchy sequence. For all $n, m\geq 0$ and $t\in[0,T]$, from (8), (H3) and Step 1, we have

$$\begin{aligned}
&E\sup_{0\le s\le t}\|x^{n+1}(s)-u(s,x_s^{n+1})-x^{m+1}(s)+u(s,x_s^{m+1})\|_X^2 \le 3E\sup_{0\le s\le t}\left\|\int_0^s AS(s-r)[u(r,x_r^{n+1})-u(r,x_r^{m+1})]\mathrm{d}r\right\|_X^2\\
&\quad+3E\sup_{0\le s\le t}\left\|\int_0^s S(s-r)[f(r,x_r^{n})-f(r,x_r^{m})]\mathrm{d}r\right\|_X^2+3E\sup_{0\le s\le t}\left\|\int_0^s S(s-r)[g(r,x_r^{n})-g(r,x_r^{m})]\mathrm{d}w(r)\right\|_X^2\\
&\le 3K_0^2M_{1-\alpha}^2a^{-2\alpha}\Gamma(2\alpha-1)E(\sup_{0\le s\le t}\|x^{n+1}(s)-x^{m+1}(s)\|_X^2)+3(T+C_4)\int_0^t G(r,E(\sup_{0\le u\le r}\|x^n(u)-x^m(u)\|_X^2))\mathrm{d}r, \quad (15)
\end{aligned}$$

where C_4 is a generic constant used by Liu [1, Theorem 1.2.6, p. 14]. Therefore,

$$\begin{aligned}
E(\sup_{0\le s\le t}\|x^{n+1}(s)-x^{m+1}(s)\|_X^2) &\le \frac{1}{1-K_0\|(-A)^{-\alpha}\|}E\sup_{0\le s\le t}\|x^{n+1}(s)-u(s,x_s^{n+1})-x^{m+1}(s)+u(s,x_s^{m+1})\|_X^2\\
&\quad+\frac{1}{K_0\|(-A)^{-\alpha}\|}E\sup_{0\le s\le t}\|u(s,x_s^{n+1})-u(s,x_s^{m+1})\|_X^2\\
&\le \frac{3K_0^2M_{1-\alpha}^2a^{-2\alpha}\Gamma(2\alpha-1)}{1-K_0\|(-A)^{-\alpha}\|}E(\sup_{0\le s\le t}\|x^{n+1}(s)-x^{m+1}(s)\|_X^2)\\
&\quad+\frac{3(T+C_4)}{1-K_0\|(-A)^{-\alpha}\|}\int_0^t G(r,E(\sup_{0\le u\le r}\|x^n(u)-x^m(u)\|_X^2))\mathrm{d}r\\
&\quad+K_0\|(-A)^{-\alpha}\|E(\sup_{0\le s\le t}\|x^{n+1}(s)-x^{m+1}(s)\|_X^2). \quad (16)
\end{aligned}$$

Let

$$z(t):=\limsup_{n,m\to\infty}E(\sup_{0\le s\le t}\|x^n(s)-x^m(s)\|_X^2).$$

By (7), assumption (H3-b) and the Fatou lemma, we have

$$z(t)\le C_5\int_0^t G(s,z(s))\mathrm{d}s,$$

where $C_5=3(T+C_4)/[(1-\gamma)(1-K_0\|(-A)^{-\alpha}\|)]$. By assumption (H3-b) we obtain $z(t)=0$. This shows that $\{x^n(t),\ n\geq 0\}$ is Cauchy.

Step 3. We claim the existence and uniqueness of the solution to Eq. (1). Borel–Cantelli lemma shows that, as $n\to\infty$, $x^n(t)\to x(t)$ holds uniformly for $0\le t\le T$. Hence, taking limits on both sides of (8), we obtain that $x(t)$ is a solution to Eq. (1). This shows the existence. And the uniqueness of the solutions could be obtained by the same procedure as step 2. The proof is complete. □

Remark 3.1. If $G(t,u)=K_1u$ for some constant K_1, then condition (H3) implies a global Lipschitz condition, which is studied in [2]. If, in Remark 2.1, $L(t)=1$, the condition is studied in [3]. Therefore, some previous results [2,3] are improved and generalized.

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